國立聯合大學104學年度

_____(院)系寒假轉學生招生考試試題紙

科目：微 積 分　　第 1 頁 共 2 頁

一.單選題(1–10題，每題 3 分)

1. $\frac{d}{dx}(\int_0^x \sqrt{t^3+1}\,dt) =$ (A) 0; (B) $\frac{3x^2}{2\sqrt{x^3+1}}$; (C) $3x^2\sqrt{x^3+1}$; (D) $\frac{1}{2\sqrt{x^3+1}}$; (E) $\sqrt{x^3+1}$.

2. If $\int_0^5 f(x)\,dx = 7$ and $\int_2^5 f(x)\,dx = 4$, then $\int_2^0 f(x)\,dx =$ (A) 0; (B) -11. (C) 11; (D) -3; (E) 3;

3. If $f(x,y) = 1 - x^3y^2 + xy + 2y$, then $\frac{\partial f}{\partial x} =$

   (A) $1 - 3x^2y^2 + y$; (B) $1 - 3x^2y^2 + y + 2y$; (C) $-3x^2y^2 + y$; (D) $-3x^2y^2 + y + 2y$; (E) $-6x^2y + 1 + 2y$.

4. If a, b and c are constants(常數), which of the following partial fraction decomposition of $\frac{5x^2-2x+5}{x(x-2)^2}$ is true?

   (A) $\frac{a}{x(x-2)} + \frac{b}{x(x-2)^2}$; (B) $\frac{a}{x} + \frac{b}{x-2}$; (C) $\frac{a}{x} + \frac{bx+c}{x-2}$; (D) $\frac{a}{x} + \frac{b}{x-2} + \frac{c}{(x-2)^2}$; (E) $\frac{a}{x} + \frac{b}{(x-2)^2}$.

5. Which of the following statements is **NOT** correct?

   (A) $\lim_{x\to\infty}(\ln(x-1) - \ln(x^2+1)) = -\infty$; (B) $\lim_{x\to\infty}\frac{e^{2x}-1}{e^x-e^{-x}} = \infty$; (C) $\lim_{x\to-\infty} e^x = 0$; (D) $\lim_{x\to\infty}\ln x = \infty$;

   (E) $\lim_{x\to\infty}\frac{1-3x}{x-1} = 3$.

6. Let $f(x) = \begin{cases}\sqrt{3x+1}, & \text{if } x > 1\\ 4x-2, & \text{if } x < 1.\end{cases}$ Choose the **WRONG** statement about f.

   (A) f is discontinuous at $x = 1$; (B) the domain of f is $\{x|x \neq 1\}$; (C) the limit $\lim_{x\to1} f(x)$ does not exist;

   (D) $\lim_{x\to1^+} f(x) = 2$; (E) $\lim_{x\to1^+} f(x) = \lim_{x\to1^-} f(x)$.

7. If $f(x) = (\ln x)^2$, then $f'(x) =$ (A) $2\ln x$; (B) $\frac{2(\ln x)}{x}$; (C) $\frac{1}{2x}$; (D) $\frac{2}{x}$; (E) $(\frac{1}{x})^2$.

8. If $f(x) = \sqrt{\sin x}$, then $f'(x) =$ (A) $\frac{1}{2\sqrt{\cos x}}$; (B) $\frac{1}{2\sqrt{-\cos x}}$; (C) $\frac{\cos x}{2\sqrt{\sin x}}$; (D) $\frac{-\cos x}{2\sqrt{\sin x}}$; (E) $\sqrt{\cos x}$.

9. If $f(x) = xe^{3x}$, then $f'(x) =$ (A) $3e^{3x}$; (B) $3xe^{3x}$; (C) $(1+3x)e^{3x}$; (D) $e^{3x} + 3x^2e^{3x-1}$; (E) $3xe^{3x-1}$.

10. $\int \frac{x-1}{\sqrt{x}}\,dx =$ (A) $\frac{2x^{3/2}}{3} - 2x^{1/2} + C$; (B) $\frac{x^{-1/2}}{2} + \frac{x^{-3/2}}{2} + C$; (C) $\frac{\frac{x^2}{2}-x}{\frac{2x^{3/2}}{3}} + C$; (D) $2x^{1/2}\left(\frac{x^2}{2} - x\right) + C$;

    (E) $\ln x + \frac{1}{x} + C$.

二.單選題(11–20題，每題 5 分)

11. $\int_0^1 \frac{x-1}{x+1}\,dx =$ (A) 0; (B) 1; (C) $\frac{-1}{3}$; (D) $1 - 2\ln 2$; (E) $\frac{-\ln 2}{2}$.

12. Using integration by parts, we have $\int x\ln(2x+1)\,dx = \frac{x^2}{2}\ln(2x+1) - \int f(x)\,dx$, where $f(x) =$

    (A) $\frac{1}{2x+1}$; (B) $\frac{2}{2x+1}$; (C) $\frac{x^2}{4x+2}$; (D) $\frac{x^2}{2x+1}$; (E) $\frac{2x^2}{2x+1}$.

13. Let R be the region in the xy-plane enclosed by $y = \sqrt{x}$, $y = 0$ and $x = 4$, then $\iint_R f(x,y)\,dA =$

    (A) $\int_0^4\int_0^2 f(x,y)\,dy\,dx$ (B) $\int_0^2\int_0^{y^2} f(x,y)\,dx\,dy$; (C) $\int_0^{y^2}\int_0^2 f(x,y)\,dy\,dx$; (D) $\int_0^{\sqrt{x}}\int_0^4 f(x,y)\,dx\,dy$;

    (E) $\int_0^4\int_0^{\sqrt{x}} f(x,y)\,dy\,dx$.

14. Given $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, then $\sin(x^2) =$

(A) $\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}\right)^2$; (B) $\left(\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}\right)^2$; (C) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)!}$; (D) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)!}$; (E) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$.

15. Let $f(x,y) = x^3y^2$. Find the gradient vector $\nabla f(2,1)$ of f at $(2,1)$.

(A) $6\vec{\mathbf{i}} + 8\vec{\mathbf{j}}$; (B) $6\vec{\mathbf{i}} - 16\vec{\mathbf{j}}$; (C) $6\vec{\mathbf{i}} + 16\vec{\mathbf{j}}$; (D) $12\vec{\mathbf{i}} + 6\vec{\mathbf{j}}$; (E) $12\vec{\mathbf{i}} + 16\vec{\mathbf{j}}$.

16. $I = \int \frac{x}{(x^2+1)^3}\, dx$. If we let $u = x^2 + 1$, then $I =$

(A) $\int \frac{\sqrt{u-1}}{u^3}\, du$; (B) $\int \frac{\sqrt{u-1}}{2u^3}\, du$; (C) $\int \frac{1}{2u^3}\, du$; (D) $\int \frac{1}{u^3}\, du$; (E) $\int \frac{u-1}{2u^3}\, du$.

17. $\int_0^1 \int_1^2 (4x^3y + x)\, dy\, dx =$

(A) $\int_0^1 6x^3\, dx$; (B) $\int_0^1 (x^4 + 3/2)\, dx$; (C) $\int_0^1 (6x^3 + 3x)\, dx$; (D) $\int_0^1 (6x^3 + x)\, dx$; (E) $\int_0^1 x^4\, dx$.

18. If $z = 4xy^2 + x^3$, $x = 2u + v$, $y = 3u - 2v$, then $\partial z/\partial u =$

(A) $(4y^2 + 3x^2)(1) + (8xy)(-2)$;
(B) $(4y^2 + 3x^2)(1) + (8xy + x^3)(-2)$;
(C) $(4y^2 + 3x^2)(3) + (8xy)(2)$;
(D) $(4y^2 + 3x^2)(2) + (8xy)(3)$;
(E) $(4y^2 + 3x^2)(2) + (8xy + x^3)(3)$.

19. If $x^2y^3 + 3x = 4y$, then $\frac{dy}{dx} =$

(A) $\frac{3}{4 - 6xy^2}$; (B) $\frac{3}{4 - 3x^2y^2}$; (C) $\frac{2xy^3 + 3}{4 - 3x^2y^2}$; (D) $\frac{4 - 3x^2y^2}{2xy^3 + 3}$; (E) $-\frac{2xy^3 + 3 - 4y}{3x^2y^2 + 3x - 4}$.

20. If $f(x) = 6x^5 - 5x^6$. Which statement is **WRONG**?

(A) The critical numbers of f are 0 and 1;
(B) f is decreasing on $(1, \infty)$;
(C) f is increasing on $(0, 1)$;
(D) $f(1)$ is a local maximum value of f;
(E) $f(0)$ is a local minimum value of f.

三.是非題: 下列各題的敘述若是正確請選(A)，若是錯誤請選(B)(21–30題，每題 2 分)

21. $e^{2\ln 3} = 6$.

22. $\frac{d}{dx} \ln 3 = \frac{1}{3}$.

23. $x = 0$ is a vertical asymptote of $y = \ln x$.

24. $\int_0^3 \int_0^2 x^3 y\, dx\, dy = \left(\int_0^2 x^3\, dx\right)\left(\int_0^3 y\, dy\right)$.

25. If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then $\lim_{x\to a}(f(x) - g(x)) = 0$.

26. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ is convergent.

27. The inverse function of $y = e^{3x}$ is $y = (\ln x)/3$.

28. There is a function such that $f(x) < 0$, $f'(x) < 0$ and $f''(x) > 0$ for all $x \in \mathbb{R}$.

29. If $(2,1)$ is a critical point of $f(x,y)$ and $\left(\frac{\partial^2 f}{\partial x^2}(2,1)\right)\left(\frac{\partial^2 f}{\partial y^2}(2,1)\right) < \left(\frac{\partial^2 f}{\partial x \partial y}(2,1)\right)^2$, then $(2,1)$ is a saddle point of f.

30. If $f(x,y)$ has a local maximum at (a,b) , then $\nabla f(a,b) = \vec{0}$.