

# 國立聯合大學 104 學年度

(院)學系轉學生招生考試試題紙

科目： 工程數學 第 1 頁共 1 頁

1. [10%] 請求解右列微分方程式：
$$\frac{dy}{dx} = \frac{x/y}{e^{x^2+y^2} + e^{x^2-y^2}}.$$

2. [10%] 請求解右列微分方程式：
$$y'' + 3y' + 2y = \cosh x + \sinh 2x.$$

3. 有一函數是  $f(t) = \begin{cases} t, & 0 \leq t < 2 \\ 2, & 2 \leq t < 4 \\ 6-t, & 4 \leq t < 6 \\ 0, & \text{elsewhere} \end{cases}$ . [5%](a) 請繪出函數  $f(t)$  之圖形. [5%](b) 請求

出其拉普拉斯轉換(Laplace transform).

4. [10%] 請求解右列積分方程式：
$$f(t) = e^{-2t} \sin t + \int_0^t f(t-\tau) d\tau$$

5. [10%] 請求出  $x(t) = \mu(t) - \mu(t-5)$  與  $h(t) = \mu(t) - \mu(t-3)$  之摺積積分 (convolution integral), 其中  $\mu(t)$  是單位步級函數.

6. [10%] Consider the curve  $C$  which position vector is  $\mathbf{r}(t)$ . If the second derivative  $\mathbf{r}''(t) = 2\mathbf{j} + 2t\mathbf{k}$  and  $\mathbf{r}'(0) = 2\mathbf{i}$ , find the length of the curve  $C$  on the interval  $-1 \leq t \leq 1$ .

7. [10%] Consider the function  $F(x, y, z) = xy \cos yz$  and the point  $P(1, 2, 0)$ . Find the directional derivative of  $F(x, y, z)$  at  $P$  in the direction of the origin. Also find a unit vector normal to the surface  $xy \cos yz = 2$  at  $P$ .

8. [10%] Evaluate  $\oint_C 2y dx + 5x dy$ , where  $C$  is the circle  $(x-1)^2 + (y+3)^2 = 25$ .

9. [10%] Find  $\oint_C \mathbf{F} \cdot d\mathbf{r}$ , where  $\mathbf{F} = y^3 \mathbf{i} - x^3 \mathbf{j} + z^3 \mathbf{k}$  and  $C$  is the trace of the cylinder  $x^2 + y^2 = 1$  in the plane  $x + y + z = 1$ .

10. [10%] If  $\mathbf{F} = 4x\mathbf{i} + y\mathbf{j} + 4z\mathbf{k}$ , evaluate the outward flux  $\iint_S (\mathbf{F} \cdot \mathbf{n}) dS$ , where  $S$  is the sphere  $x^2 + y^2 + z^2 = 4$ .