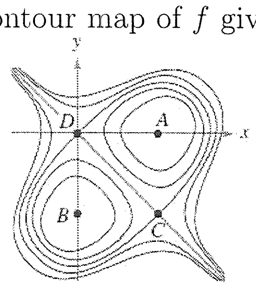
國立聯合大學105學年度

暑假轉學生招生考試試題紙

科目： 微 積 分

一.是非題: 在下列各題中，若敘述是正確的請答(A)，若是錯誤的請答(B)(1–5題，每題 2 分)

1. The contour map of f given below shows that the point A is a saddle point of f.



2. From the contour map of f given above, we have that the gradient vector of f at point C is $< 0, 0 >$.
3. $(\ln x)^3 = 3\ln x$.
4. $\displaystyle\int_0^3\int_0^{3x}\cos(y^2)\,dy\,dx = \int_0^9\int_{y/3}^3\cos(y^2)\,dx\,dy$.
5. The inverse function of $f(x) = x^3$ is $g(x) = \dfrac{1}{x^3}$.

二.單選題(6–15題，每題 3 分)

6. If $f(x) = \sin^2 x$, then $f'(x) =$ (A) $2\sin x$; (B) $\sin(2x)$; (C) $2\cos x$; (D) $\cos^2 x$; (E) $-2\sin x\cos x$.
7. If $f(x) = \sqrt{2x+3}$, then $f'(3) =$ (A) $\dfrac{1}{3}$; (B) $\dfrac{1}{6}$; (C) $\dfrac{1}{2\sqrt{2}}$; (D) 1; (E) $\dfrac{2}{3}$.
8. If $f(x) = e^{5x}$, then $f'(x) =$ (A) $5xe^{5x-1}$; (B) $5xe^{5x}$; (C) $5e^{5x}$; (D) e^{5x}; (E) $\dfrac{e^{5x}}{5}$.
9. If $y = \ln|1-x|$, then $y' =$ (A) $\dfrac{1}{\ln|1-x|}$; (B) $\dfrac{1}{|1-x|}$; (C) $\dfrac{-1}{|1-x|}$; (D) $\dfrac{1}{1-x}$; (E) $\dfrac{1}{x-1}$.
10. If $\displaystyle\int_0^7 f(x)\,dx = 10$ and $\displaystyle\int_3^7 f(x)\,dx = 8$, then $\displaystyle\int_0^3 f(x)\,dx =$ (A) 2; (B) -2; (C) $5/4$; (D) $-4/5$; (E) 18.
11. $\dfrac{d}{dx}\left(\displaystyle\int_0^x e^{t^2}\,dt\right) =$ (A) $\dfrac{e^{t^2}}{2t}$; (B) $\dfrac{e^{x^2}}{2x}$; (C) $2xe^{x^2}$; (D) e^{x^2}; (E) e^{t^2}.
12. If $f(x,y) = x^2y^3 + 3y\sqrt{x} - x$, then $\dfrac{\partial f}{\partial y} =$

    (A) $x^2y^2 + 3\sqrt{x}$; (B) $3x^2y^2 + 3\sqrt{x} - x$; (C) $3y^2 + 3\sqrt{x}$; (D) $3x^2y^2 + 3\sqrt{x}$; (E) $2xy^3 + \dfrac{3y}{2\sqrt{x}} - 1$.
13. If $f'(x) = x^2(2-x)$, then which of the following statements is true?

    (A) $f(0)$ is a local maximum value of f; (B) $f(0)$ is a local minimum value of f;
    (C) $f(2)$ is a local minimum value of f; (D) $f(2)$ is neither a local maximum nor a local minimum of f;
    (E) $f(2)$ is an absolute maximum value of f.
14. Given $f(x) = \displaystyle\sum_{n=0}^{\infty}(n+1)^2x^{2n}$, $|x| < 1$, then $f'(x) =$

    (A) $\displaystyle\sum_{n=0}^{\infty}\frac{(n+1)^2}{2n+1}x^{2n+1}$; (B) $\displaystyle\sum_{n=0}^{\infty}\frac{(n+1)^3}{3}x^{2n}$; (C) $\displaystyle\sum_{n=1}^{\infty}4n(n+1)x^{2n-1}$; (D) $\displaystyle\sum_{n=0}^{\infty}2(n+1)x^{2n}$; (E) $\displaystyle\sum_{n=1}^{\infty}2n(n+1)^2x^{2n-1}$.
15. If $x^2y = \sin(xy)$ defines y as a differentiable function of x, then $\dfrac{d}{dx}(x^2y) =$

    (A) $2x\dfrac{dy}{dx}$; (B) $2xy\dfrac{dy}{dx}$; (C) $2xy + x^2\dfrac{dy}{dx}$; (D) $2xy + x^2y\dfrac{dy}{dx}$; (E) $2xy + x^2$.

二.單選題(16–27題，每題 5 分)

16. Let $f(x)=\begin{cases}\dfrac{x-2}{x^2-4}, & \text{if } x\neq 2\\ 3, & \text{if } x=2.\end{cases}$ Choose the correct statement about f.

(A) $\lim_{x\to 2} f(x)=3$; (B) f is continuous on $\{x|x\neq 2\}$; (C) the domain of f is $\mathbb{R}$; (D) $\lim_{x\to 2^+} f(x)\neq \lim_{x\to 2^-} f(x)$;
(E) $\lim_{x\to 2} f(x)$ does not exist.

17. Using integration by parts, we have $\int x^2\cos(3x)\,dx = x^2 g(x)-\int f(x)\,dx$, then $f(x)=$

(A) $6x\sin(3x)$; (B) $-6x\sin(3x)$; (C) $\frac{1}{3}\sin(3x)$; (D) $\frac{2}{3}x\sin(3x)$; (E) $\frac{1}{3}x^2\sin(3x)$.

18. Let R be the region in the xy-plane enclosed by $y=x^3$, $y=8$ and $x=0$, then $\iint_R f(x,y)\,dA=$

(A) $\int_0^2\int_0^8 f(x,y)\,dy\,dx$ (B) $\int_0^{y^{1/3}}\int_0^8 f(x,y)\,dy\,dx$; (C) $\int_0^8\int_{y^{1/3}}^2 f(x,y)\,dx\,dy$; (D) $\int_0^2\int_{x^3}^8 f(x,y)\,dy\,dx$;
(E) $\int_0^2\int_0^{x^3} f(x,y)\,dy\,dx$.

19. If $f^{(n)}(0)=\frac{3^n}{(n+1)}$, for all $n=0,1,2,...$, then the Taylor series of f about 0 is

(A) $\sum_{n=0}^{\infty}\frac{3^n}{(n+1)}x^n$; (B) $\sum_{n=0}^{\infty}\frac{3^n}{(n+1)}x^{n+1}$; (C) $\sum_{n=0}^{\infty}\frac{3^n}{(n+1)!}x^n$; (D) $\sum_{n=1}^{\infty}\frac{3^n}{(n+1)(n-1)!}x^{n-1}$; (E) $\sum_{n=0}^{\infty}\frac{3^n}{(n+1)!}x^{n+1}$.

20. If the maximum rate of change of $f(x,y)$ at (a,b) is 2 and it occurs in the direction of the vector $<1,1>$, then the gradient vector $\nabla f(a,b)=$

(A) $<2,2>$; (B) $<-2,-2>$; (C) $<\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}>$; (D) $<-\sqrt{2},-\sqrt{2}>$; (E) $<\sqrt{2},\sqrt{2}>$.

21. $I=\int x(3x+1)^{10}\,dx$. If we let $u=3x+1$, then $I=$

(A) $\int\frac{u^{10}}{3}\,du$; (B) $\int\frac{1}{3}u^{11}\,du$; (C) $\int\frac{1}{3}(u^{11}-u^{10})\,du$; (D) $\int\frac{1}{9}(u^{11}-u^{10})\,du$; (E) $\int(u^{11}-u^{10})\,du$.

22. $\frac{x^2+2}{x(x-1)^2}=\frac{a}{x}+\frac{b}{x-1}+\frac{c}{(x-1)^2}$, where a,b and c are constants(常數), then $a+b+c=$

(A) -1; (B) 6; (C) 4; (D) 2; (E) 0.

23. $\int_0^1\int_1^y(2x+y)\,dx\,dy=\int_0^1 f(x,y)\,dy$, where $f(x,y)=$

(A) $2y^2-y-1$; (B) $2y^2+y-1$; (C) $2y^2+y+1$; (D) y^2-1; (E) y^2+1.

24. Given $z=f(s^2+2t,3s-t)$, $\frac{\partial f}{\partial x}(1,1)=4$, $\frac{\partial f}{\partial y}(1,1)=2$, $\frac{\partial f}{\partial x}(3,2)=7$ and $\frac{\partial f}{\partial y}(3,2)=-5$, find $\frac{\partial z}{\partial s}|_{(s,t)=(1,1)}$.

(A) -8; (B) -1; (C) 14; (D) 18; (E) 20.

25. Let $f(x,y)=2x^3+xy^2+5x^2+y^2-4$. Which statement is correct?

(A) $(0,0)$ is a saddle point of f; (B) $f(-5/3,0)$ is a local minimum vlaue of f;
(C) f has no local maximum values; (D) $f(1,1)$ is a local minimum value of f;
(E) $(-1,2)$ is a saddle point of f.

26. Using polar coordinates, we have $\int_{-2}^2\int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\,dy\,dx=$

(A) $\int_0^2\int_{-\pi/2}^{\pi/2} e^{r^2}\,dr\,d\theta$; (B) $\int_0^2\int_{-\pi/2}^{\pi/2} re^{r^2}\,dr\,d\theta$; (C) $\int_{-2}^2\int_0^{\pi} re^{r^2}\,dr\,d\theta$; (D) $\int_0^2\int_0^{\pi} e^{r^2}\,dr\,d\theta$; (E) $\int_0^2\int_0^{\pi} re^{r^2}\,dr\,d\theta$.

27. The tangent plane to the surface $x^2yz=z^3-x$ at $(1,2,-1)$ is

(A) $2(x-1)-(y-2)-3(z+1)=0$; (B) $3(x-1)-(y-2)-(z+1)=0$; (C) $3(x-1)+(y-2)+(z+1)=0$;
(D) $(x+3)+2(y+1)-(z+1)=0$; (E) $(x-3)+2(y+1)-(z+1)=0$.