國立聯合大學 105 學年度碩士班考試招生

電子工程學系碩士班 入學考試試題

科　目：工程數學　　第 1 頁共 1 頁

1. (10%) Given second order linear differential equation:

$$y'' - 3y' + 2y = e^x, \quad y(0) = 1 \text{ and } y'(0) = 1$$

(a) Find general solution

(b) Find final solution

2. (10%) Please solve $\dfrac{dy}{dx} = \dfrac{-2xy^3 - 2}{3x^2y^2 + e^y}$

3. (10%) Please solve $\dfrac{dy}{dx} + y = y^6$

4. (10%) Please solve ODEs:

(a) $x^2y'' - 4xy' + 4y = 0$

(b) $x^2y'' + xy' - y = 2x^3$

5. (10%) Please solve the following equations, where $\delta(t)$ means the unit impulse.

(a) $y'' + 7y' + 10y = \delta(t),\ y(0) = 0,\ y'(0) = 0$

(b) $y(t) = e^{-(t-t_0)}u(t-t_0) + \int_0^t y(t-\tau)d\tau$

6. (10%) Please solve the linear system $\begin{cases} x' - 2y' = 1 \\ x' - x + y = 0 \end{cases}$; $x(0) = y(0) = 0$

7. (10%) Let $f(t) = e^{-|t|}$, $-\infty < t < \infty$, please solve the following questions.

(a) Find its Fourier transform.

(b) Find the Fourier transform of $f(t-5)$

8. (10%) Let $f(t) = u(t+4) - u(t+2) + u(t-2) - u(t-4)$ and $u(t)$ means the unit step function, please solve the following questions.

(a) Find its Fourier transform.

(b) Evaluate the amplitude spectrum and the phase spectrum carefully.

9. (10%) Use the theory of vectors to find the straight line L_1 through the point P: (2, 3) in the *xy*-plane and perpendicular to the straight line L_2: $x + 2y = 2$.

10. (10%) If vectors $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{v} \cdot \mathbf{w} = 0$, does it imply that vectors $\mathbf{u} \cdot \mathbf{w} = 0$? Prove or disprove. *HINT*: If it is true, it needs to be proved in general, i.e., for all possible cases. But if it is false, it can be disproved merely by putting forward a single counterexample.