國立聯合大學105學年度

______________(院)系寒假轉學生招生考試試題紙

科目：微積分　　第 1 頁 共 2 頁

一.單選題(1–10題，每題 3 分)

1. If $y = \ln(e^{2x} + x^3)$, then $y' =$ (A) $\dfrac{1}{\ln(e^{2x}+x^3)}$; (B) $2 + \dfrac{1}{x^3}$; (C) $2 + \dfrac{3}{x}$; (D) $\dfrac{e^{2x}+3x^2}{e^{2x}+x^3}$; (E) $\dfrac{2e^{2x}+3x^2}{e^{2x}+x^3}$.

2. If $f(x) = (\sin x)^2$, then $f'(\pi/3) =$ (A) $\sqrt{3}$; (B) $\sqrt{3}/2$; (C) 1; (D) 1/4; (E) 0.

3. If $f(x) = xe^{x^2}$, then $f'(x) =$ (A) e^{x^2}; (B) $2xe^{x^2}$; (C) $(1+2x^2)e^{x^2}$; (D) $(1+x)e^{x^2}$; (E) $e^{x^2} + x^3e^{x^2-1}$.

4. If $f(x) = \dfrac{1}{\sqrt{x}+1}$, then $f'(4) =$ (A) 1/18; (B) 1/36; (C) −1/36; (D) −1/18; (E) −2/9.

5. Let $F(x) = \int_5^x \sin(t^2)\,dt$, then $F'(x) =$ (A) $\sin(x^2)/(2x)$; (B) $\sin(t^2)$; (C) $2t\sin(t^2)$; (D) $\sin(x^2)$; (E) $2x\sin(x^2)$.

6. If $\int_3^8 f(x)\,dx = 4$ and $\int_2^8 f(x)\,dx = 9$, then $\int_2^3 f(x)\,dx =$ (A) 5; (B) −5. (C) 11; (D) −11; (E) 36;

7. If $f(x,y) = x^3 + xy^2 + 4y$, then $\dfrac{\partial f}{\partial x}(2,-1) =$ (A) 12; (B) 8; (C) 13; (D) 17; (E) 9.

8. If $\int_0^1 f(x)\,dx = 3$, then $\int_0^1 xf(x^2)\,dx =$ (A) 3/2; (B) 3; (C) 9; (D) 6; (E) −3.

9. If $f'(3) = 5$, then $\lim_{h\to 0} \dfrac{f(3+2h)-f(3)}{h} =$ (A) 10; (B) 5; (C) −5; (D) 25; (E) 5/2.

10. $\lim_{n\to\infty} \dfrac{e^x}{x^{100}} =$ (A) 0; (B) 1; (C) e; (D) $e/(100!)$; (E) ∞.

二.單選題(11–22題，每題 5 分)

11. Let $f(x) = \begin{cases} \dfrac{x-2}{x^2-4}, & \text{if } x \neq 2 \\ 0, & \text{if } x = 2. \end{cases}$ Choose the **WRONG** statement about f.
    - (A) $f(2) = 0$;
    - (B) f is discontinuous at $x = 2$;
    - (C) f has only one discontinuity;
    - (D) $\lim_{x\to 2} f(x) = 1/4$;
    - (E) $\lim_{x\to 1} f(x) = 1/3$.

12. $\int_1^e \dfrac{x^2+1}{x}\,dx =$ (A) $2e/3 + 2/e - 8/3$; (B) $(e^2+3)/2$; (C) $e^2 + 1/2$; (D) $(e^2+1)/2$; (E) $(e^2-1)/2$.

13. The slope of the tangnent line to $x^3 + 2xy^2 = 5x + 4$ at $(1,2)$ is
    (A) −1/2; (B) −1/4; (C) 1/4; (D) −3/4; (E) 3/4.

14. Using integration by parts, we have $\int_0^1 xe^{3x}\,dx = \dfrac{e^3}{3} - \int_0^1 f(x)\,dx$, where $f(x) =$
    (A) e^{3x}; (B) $\dfrac{3x^2e^{3x}}{2}$; (C) $3e^{3x}$; (D) $\dfrac{e^{3x}}{3}$; (E) $\dfrac{e^{3x+1}}{3x+1}$.

15. $\int_0^y (3x^2y + y^2)\,dx =$ (A) $(3/2)y^4$; (B) $(3/2)y^4 + (1/3)y^3$; (C) y^4; (D) $y^3 + y^4$; (E) $x^3y + xy^2$.

16. If $f'(x) = x(x-1)^2$. Which statement is**WRONG**?

(A) $f(1)$ is a local maximum value of f;

(B) $f(0)$ is a local minimum value of f;

(C) $f(0)$ is an absolute maximum value of f;

(D) f is decreasing on $(-\infty, 0)$;

(E) The x-coordinates of the inflection points are $1/3$ and 1.

17. The directional derivative of $f(x,y) = x^2 + y^2$ at (2,1) in the direction of $\vec{v} = 3\vec{i} - 4\vec{j}$ is

(A) $(12/5)\vec{i} + (8/5)\vec{j}$; (B) $(12/5)\vec{i} - (8/5)\vec{j}$; (C) $12\vec{i} - 8\vec{j}$; (D) 4; (E) 4/5.

18. If we let $u = \sqrt{x^2+1}$, then $\int \frac{x^3}{\sqrt{x^2+1}}\,dx =$

(A) $\int \frac{(\sqrt{u^2-1})^{3/2}}{u}\,du$; (B) $\int (u^3 - u^2)\,du$; (C) $\int (u^2-1)\,du$; (D) $\int 2\sqrt{u^2-1})^{3/2}\,du$; (E) $\int \frac{u^2-1}{u}\,du$.

19. If $z = f(x,y)$, $\frac{\partial f}{\partial x} = 2x + 3y$, $\frac{\partial f}{\partial y} = 3x + 4$, $x = u^2 + v$ and $y = -2u + v^2$, then $\frac{\partial z}{\partial u}|_{(1,2)} =$

(A) $-2x + 6y - 8$; (B) $5x + 3y + 4$; (C) 76; (D) -2; (E) 25.

20. Let R be the region in the xy-plane enclosed by $y = \sqrt{x}$, $y = 0$ and $x = 4$ then $\iint_R f(x,y)\,dA =$

(A) $\int_0^2 \int_0^4 f(x,y)\,dx\,dy$

(B) $\int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dx\,dy$;

(C) $\int_0^2 \int_0^{y^2} f(x,y)\,dx\,dy$;

(D) $\int_0^2 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$;

(E) $\int_0^2 \int_{y^2}^4 f(x,y)\,dx\,dy$.

21. Given $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, then $\cos(3x) =$

(A) $\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}\right)^3$; (B) $\sum_{n=0}^{\infty} \frac{(-1)^n 3x^{2n+1}}{(2n+1)!}$; (C) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)!}$; (D) $\sum_{n=0}^{\infty} \frac{(-1)^n 3x^{2n}}{(2n)!}$; (E) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n}}{(2n)!}$.

22. Which of the following integral represents the area enclosed by the curves $y = 2 - x^2$ and $y = x$?

(A) $\int_{-2}^{1} x\,dx$; (B) $\int_{-2}^{1} (2 + x - x^2)\,dx$; (C) $\int_{-2}^{1} (2 - x^2)\,dx$; (D) $\int_{-2}^{1} (x^2 + x - 2)\,dx$; (E) $\int_{-2}^{1} (2 - x - x^2)\,dx$.

三.是非題: 下列各題的敘述若是正確請選(A)，若是錯誤請選(B)(23–27題，每題 2 分)

23. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}\right)^2 + 5\right)\frac{1}{n} = \int_0^2 (x^2+1)\,dx$.

24. $x = 2$ is a vertical asymptote of $y = \frac{x^2 - x - 2}{x^2 - 4}$.

25. If $f(x) > g(x)$ for all $x \in \mathbb{R}$, then $\lim_{x\to 0} f(x) > \lim_{x\to 0} g(x)$.

26. $\lim_{x\to -\infty} e^x = 0$.

27. If $f(x)$ is continuous on $[0,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_0^{\infty} f(x)\,dx$ is convergent.