國立聯合大學 106 學年度碩士班考試招生

電子工程學系碩士班　入學考試試題

科　目：工程數學　　第 1 頁共 1 頁

1. (12%) Please solve the following ODEs:

   (a) $\dfrac{dy}{dx} = -\dfrac{2xy^3 + 2}{3x^2y^2 + e^y}$　　(b) $\cos(x+y)dx + \left(3y^2 + 2y + \cos(x+y)\right)dy = 0$

2. (10%) Given second order linear differential equation:

   $y'' - 3y' + 2y = x^2 + 1$, $y(0) = 1$ and $y'(0) = 1$

   (a) Find general solution

   (b) Find final solution

3. (8%) Please solve $x\dfrac{dy}{dx} + y = -2xy^2$

4. (10%) Find the general solution of $x^2y'' - 4xy' + 6y = 4$

5. (15%) Given $\mathbf{A} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = \mathbf{i} - \mathbf{j} - \mathbf{k}$,

   evaluate (a) $3\mathbf{A} - 2\mathbf{B}$

   (b) $\mathbf{A} \bullet \mathbf{B}$ (dot product of $\mathbf{A}$ and $\mathbf{B}$)

   (c) $\mathbf{A} \times \mathbf{B}$ (cross product of $\mathbf{A}$ and $\mathbf{B}$)

6. (15%) Given $\varphi(x, y, z) = xz - yz$ and $\mathbf{F}(x, y, z) = 4z\mathbf{i} + 2y\mathbf{j} + (x - z)\mathbf{k}$,

   evaluate (a) $\nabla\varphi$ (gradient (梯度) of φ)

   (b) $\nabla \bullet \mathbf{F}$ (divergence (散度) of $\mathbf{F}$)

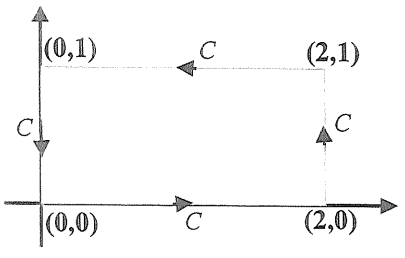
   (c) $\nabla \times \mathbf{F}$ (curl (旋度) of $\mathbf{F}$)

7. (10%) Consider the curve C with position vector $\mathbf{F}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}$ for $0 \le t \le 1$,

   evaluate (a) $\mathbf{F}'(t)$ (tangent vector to the curve C)

   (b) the length of curve C (Hint: $\int_a^b \|\mathbf{F}'(t)\| dt$)

8. (10%) Use Green's Theorem (格林定理) to evaluate $\oint_C (x^2 - 2y)dx + (2y^2 + x^2)dy$ along the path C shown below.



(Hint: Green's Theorem $\oint_C f(x,y)dx + g(x,y)dy = \iint_D \left(\dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y}\right) dA$)

9. (10%) If $f(x) = \begin{cases} 1 & \text{for } -1 \le x \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$, evaluate its Fourier integral coefficients A_ω and B_ω.

(Hint: $A_\omega = \dfrac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(\omega x)dx$　　$B_\omega = \dfrac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(\omega x)dx$)