

# 國立聯合大學 106 學年度

## 暑假轉學生招生考試試題紙

科目：微積分 第 1 頁共 2 頁

### 第一大題單選選擇題(每題 6 分共 10 題)

(1) Which of the following curves has no horizontal asymptote?

- (A)  $y = \sqrt{x^4 + x^2} - x^2$     (B)  $y = \frac{\sqrt{x^4 + x^2}}{x^2}$     (C)  $y = \frac{x^2}{\sqrt{x^4 + x^2}}$     (D)  $y = x^2 - \sqrt{x^4 + x^2}$   
(E)  $y = \sqrt{x^4 + x^2} + x^2$

(2)  $\int_0^1 2(x-1)(x^2-2x)^9 dx =$

- (A)  $\frac{1}{8}$     (B)  $\frac{1}{9}$     (C)  $-\frac{1}{12}$     (D)  $\frac{1}{10}$     (E)  $-\frac{1}{12}$

(3) Which of the following statements is correct?

- (A)  $\lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x}}{x} = 0$     (B)  $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 1$     (C)  $\lim_{x \rightarrow \infty} x \cos \frac{1}{x} = 0$     (D)  $\lim_{x \rightarrow 0} \frac{\sin \frac{1}{x}}{x} = \infty$   
(E)  $\lim_{x \rightarrow \infty} \sin \frac{1}{x}$  does not exist

(4)  $\frac{d}{dx} \int_0^x \sin t^2 dt =$

- (A) 0    (B)  $\sin x^2$     (C) 1    (D)  $\sin t^2$     (E)  $\pi$

(5) Let  $\int \sin^3 x \cos^2 x dx = A \cos^3 x + B \cos^5 x + C$ . Then  $A + B =$

- (A)  $-\frac{1}{15}$     (B)  $\frac{2}{15}$     (C)  $-\frac{2}{15}$     (D)  $\frac{1}{15}$     (E)  $-\frac{3}{15}$

(6) The length of the curve  $y = e^x$ ,  $0 \leq x \leq 1$ , is

- (A)  $\int_0^1 \sqrt{1+e^x} dx$     (B)  $\int_0^1 2\pi x \sqrt{1+e^{2x}} dx$     (C)  $\int_0^1 2\pi x \sqrt{1+e^x} dx$     (D)  $\int_0^1 \sqrt{1+e^{x^2}} dx$   
(E)  $\int_0^1 \sqrt{1+e^{2x}} dx$

(7) The radius of convergence of the series  $\sum_{n=0}^{\infty} n! (x-3)^n$  is

- (A)  $\infty$     (B)  $\frac{1}{2}$     (C)  $\pi$     (D) 0    (E) 1

(8)  $\int_0^1 \int_{3y}^3 e^{x^2} dx dy =$

- (A)  $\frac{e^9 - 1}{6}$     (B)  $\frac{e^9 - 1}{5}$     (C)  $\frac{e^6 - 1}{9}$     (D)  $\frac{e^5 - 1}{9}$     (E)  $\frac{e^6 + 1}{9}$

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科目：微積分 第 2 頁共 2 頁

(9) Which of the following points is a critical point of the function  $f(x, y) = e^{2y-x^2-y^2}$ ?

- (A) (1, 1) (B) (0, 1) (C) (1, 0) (D) (2, 1) (E) (1, 2)

(10) Given  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ ,  $|x| < 1$ , the Maclaurin series of  $\frac{1}{(1-x)^2}$  is

- (A)  $-\sum_{n=1}^{\infty} nx^n$  (B)  $-\sum_{n=0}^{\infty} (n+1)x^n$  (C)  $\sum_{n=0}^{\infty} (n+1)x^n$  (D)  $\sum_{n=1}^{\infty} (n+1)x^n$   
(E)  $\sum_{n=1}^{\infty} nx^n$

### 第二大題填充題(每題 5 分共 4 題)

- (1) If  $\int_1^2 f(x)dx = 3$  and  $\int_0^2 g(x)dx = 2$ , then  $\int_0^2 \int_1^2 [f(x) + f(x)g(y) + g(y)] dx dy = \underline{\hspace{2cm}}$
- (2) If  $z = y + f(x^2 - y^2)$ , then  $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = \underline{\hspace{2cm}}$
- (3) The absolute maximum value of  $f(x) = \frac{x}{x^2+4}$  on the interval  $[0, 3]$  is  $\underline{\hspace{2cm}}$
- (4) If  $B = [0, 1] \times [-1, 2] \times [0, 3]$ , then  $\iiint_B xyz^2 dV = \underline{\hspace{2cm}}$

### 第三大題計算題(每題 10 分共 2 題)

- (1) Find the directional derivative of the function  $f(x, y) = x^2y^3 - 4y$  at the point  $(2, -1)$  in the direction of the vector  $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$ .
- (2) Evaluate  $\iint_R (3x + 4y^2) dA$ , where  $R$  is the region in the upper half-plane bounded by the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ .